Report on the Hausdorff Trimester Program

Rigidity

September - December 2009

Organizers: Wolfgang Lück and Nicolas Monod

The main goal of this Trimester Program was to study and link *rigidity phenomena* in different areas of pure mathematics. It was therefore intrinsically an interdisciplinary enterprise within mathematics.

Rigidity occurs in pure mathematics in three flavours:

- Two objects with a rich extra structure that are equivalent in a weak sense, typically ignoring this extra structure, must be isomorphic as objects with the full structure. Morphisms that are equivalences in the weaker sense are equivalent to isomorphisms in the stronger category.

- Certain invariants of interesting objects do take values only in a small portion of the range where they are expected to live.

- All instances of some a priori broad class of structures turn out to be of a very special type, admitting a rich additional structure, sometimes leading to an unexpected complete classification.

Targets of the Trimester Program

There is a long list of rigidity phenomena whose discovery was a striking surprise and triggered a host of fruitful and lasting activities in mathematics. In many instances, these very discoveries led to outstanding open problems currently subject of intense activity. We mention three prominent instances which were the main focus of the trimester program:

- *Mostow–Margulis–Zimmer Rigidity*
  This refers to a field at the junction between Riemannian geometry, algebraic and arithmetic group theory, and ergodic theory. The general theme is to classify linear as-well as non-linear representations of

lattices in Lie groups. The prototype is the version of Mostow rigidity stating that two closed hyperbolic Riemannian manifolds of dimension ≥ 3 are isometrically diffeomorphic as soon as their fundamental groups agree. Any homotopy equivalence between two such manifolds is homotopic to an isometric diffeomorphism. There are more advanced and general situations considered by Margulis and Zimmer.

- *Rigidity in Topology*
  A fundamental open problem spurring intense activity is the *Borel Conjecture*. Two aspherical closed manifolds are conjecturally homeomorphic if their fundamental groups agree. Any homotopy equivalence of such manifolds is conjecturally homotopic to a homeomorphism A related and perhaps more specific question is the *Novikov conjecture*, which predicts the homotopy invariance of higher signatures. It turns out that this conjecture has generated (and continues to generate) a vast amount of work in geometric group theory. The latter can also be said of (a version of) the *Atiyah Conjecture*, which predicts that the L^2-Betti numbers of a closed Riemannian manifold with torsion-free fundamental group are always integers.

- *von Neumann Rigidity*
  We mention the following prominent examples. Completely general cocycle superrigidity theorems for the Bernoulli shift of a large family of groups. The question whether orbit equivalent class of groups or group actions determines already the groups or the the group action has been studied a lot. The *Free Group Factor Problem* asks whether the von Neumann algebras of two finitely generated free groups isomorphic only if the groups are isomorphic.

Contributing fields

The Trimester Program brought together experts from, used methods from and contributed substantially to the following fields:

- Algebraic K-and L-theory

- Surgery theory

- L^2-methods

- Finite von Neumann algebras and measure theory

- Geometric group theory

- Cohomological algebra

- Bounded cohomology

- Asymptotic methods on semi-simple groups

- Multiplicative ergodic theory

- Functional-analytic aspects of group representations

Special events

There were various seminars for PhD-students, where introductions to specific topics were presented by senior or by junior mathematicians. Most of the talks were decided or requested by participants on a short term notice often motivated by discussions and unsolved questions during the tea time. The nature of these talks was very informal. The workshop *Rigidity in cohomology, K-theory, geometry and ergodic theory* took place from November 23th until November 27th.

Results

There are several interesting results which were triggered or obtained during the Trimester Program and led to ongoing new activities. Here are some highlights:

- There was substantial progress on topological rigidity, namely on the Borel Conjecture.

- Manifolds with transcendental L^2-Betti numbers were constructed, a question which is closely related to the Atiyah Conjecture.

- The asymptotic behaviour of Betti numbers under towers of finite coverings which in the limit yields L^2-Betti numbers in the characteristic zero case was studied in the case of prime characteristic.

- The most substantial progress on *almost-representations* since Kazhdan's 1982 paper was obtained. This is a circle of ideas related to Ulam stability and cohomological rigidity.

- In geometric group theory, it was proved that mapping class groups have finite asymptotic dimension. This implies notably the coarse Baum–Connes Conjecture and the Novikov Conjecture for these groups.

- Buldings and their rigidity were studied; in particular, progress was made on a general rigidity conjecture for Coxeter groups.